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General Solution of Static Sphere of Perfect Fluid and Dust of Uniform Density Using Isotropic Line Element

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ABSTRACT

The general solution of a static sphere of perfect fluid of uniform density using isotropic line element has been obtained by using the additional condition of continuity at the boundary $r = a$. Here it is shown that, this solution is a solution of equation of Wyman with an additional integrating constant. If we do not put the condition of continuity at the boundary $r = a$, then it can be shown that $p \to 0$ as $k^2 \to \infty$ using equation of Wyman so the solution of static sphere of dust can be obtained using Wyman's solution by putting $p = 0$ in the solution, so that R^2 in terms ρ_0 is for dust instead of R^2 in terms of ρ, of fluid obtained by Wyman. The anomalies discussed in the present paper can be removed by new field equation. The new proposed field equation is given and it is shown that the new proposed equation can bring Newtonian approximation.

Key words: Dust, Cluster of stars, stars, Newtonian Approximation Astrophysics
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INTRODUCTION

Some anomalies discussed in earlier papers of (Rawal, Deshpande, Kelkar, & Shenoy, 1991; Rawal, Kelkar, Deshpande, & Shenoy, 1989) and Kelkar.et .al3 in 2000 and 2001 and also anomalies discussed in the present paper can be removed by new field equation. Wyman (1946) has obtained an exact solution in the case of static sphere of perfect fluid of uniform density with using the following isotropic line element.

$$ds^2 = -e^{\mu}(dr^2 + r^2 d\theta^2 + r^2 Sin^2\theta d\varphi^2) + e^{\nu} dt^2 \quad (1)$$

He has obtained the solution of (4, 4) equation as

$$e^{\mu} = \frac{4R^2}{(e^c r^2 + e^{-c})^2} \quad (2)$$

with $R^2 = 3/8\pi\rho$ and $\rho = \rho_0 + 3p$, where ρ_0 is the proper density (Eddington, 1924), e^c is an arbitrary constant, while $\rho = \rho_0 + 3p$ is regarded as a constant in his analysis. His equation (4,4) is a second order equation. The (4,4) equation is an equation which has $-8\pi T_4^4$ term on one side of Wyman equation. It will have two constants of integration in the general solution and hence the solution obtained by Wyman (1946), though exact, is only a particular solution of (4, 4) equation, since it contains only one constant of integration, namely e^c.

GENERAL SOLUTION OF (4, 4) EQUATION

It is possible to get the second constant of integration by using Lemma given below.

Lemma: If e^{μ} is a solution of the (4, 4) equation of Wyman then any constant multiple of e^{μ} is also a solution of the same equation.

Proof: Suppose

$$e^{\mu} = f(r) \quad (3)$$

then

$$\mu' e^{\mu} = f'(r) \quad (4)$$

and therefore

$$\mu' = \frac{f'(r)}{f(r)} \quad (5)$$

Now, if we put

$$e^{\mu} = kf(r) \quad \text{and} \quad e^{\mu}\mu' = kf'(r) \quad (6)$$

as we know

$$e^{\mu}(8\pi\rho) = \mu'' + \frac{\mu'^2}{4} + \frac{2\mu'}{r} \quad (7)$$

and therefore

$$\mu' = \frac{f'(r)}{f(r)} \quad (8)$$

Hence μ', μ'^2 and μ'' which occur in RHS of (4, 4) equation of Wyman are not altered by putting $e^{\mu} = kf(r)$. Therefore using this lemma we put

$$e^{\mu} = \frac{4R^2k^2}{(e^c r^2 + e^{-c})^2} \quad (9)$$

where k^2 is a numerical constant so that we now have two constants of integration namely k^2 and e^c. These two constants can be determined using continuity of $e^{\mu}$and $\partial(e^{\mu})/\partial r$ at the boundary $r = a$. Latter condition may not be necessary. This condition was suggested to us by Newtonian approximation given by Eddington in Eq (46.2) (Eddington, 1924). Eddington (1924) in Eq (46.5) has shown that the following metric

$$ds^2 = -(1 + 2\Omega)(dx^2 + dy^2 + dz^2) + (1 - 2\Omega)dt^2 \quad (10)$$

gives Newtonian approximation and Ω is approximately Newtonian potential. Continuity of e^{μ} and its derivative gives

$$e^c = \left(\frac{m}{2a^3}\right)^{\frac{1}{2}} \quad (11)$$

and

$$k = \frac{1}{R}\left[\left(\frac{a^3}{2m}\right)^{\frac{1}{2}}\left(1 + \frac{m}{2a}\right)^3\right] \quad (12)$$

Since

$$R^2 = \frac{3}{8\pi\rho} \quad (13)$$

therefore

$$k = \left(\frac{4\pi\rho a^3}{3m}\right)^{\frac{1}{2}}\left(1 + \frac{m}{2a}\right)^3 \quad (14)$$

so that $k \cong 1$. Also it can be seen that e^{ν} given by Wyman, namely

$$e^{\nu} = \left(\frac{Ar^2 + B}{e^c r^2 + e^{-c}}\right)^2 \quad (15)$$

has already two constants of integration and hence is a general solution.

RESULT AND DISCUSSION

Although we have found the value of the second constant of integration k^2 by matching $\partial(e^{\mu})/\partial r$ at $r = a$, there is nothing in Einstein's equation which makes it necessary to match the derivative of e^{μ} at $r = a$. Hence k^2 can have any numerical value, excluding negative value because e^{μ} is a potential and hence e^{μ} has to be real. In particular we can have $k^2 \to \infty$ and we can show that the pressure tends to zero using the Eq (1.1) of (Wyman, 1946).

Hence there can be a static sphere of dust according to Einstein. This is also corroborated by the solution e^{μ}, Eq(1.8) of (Wyman, 1946) which holds well for $P = 0$ because in that case $R^2 = 3/8\pi\rho_0$ instead of $R^2 = 3/8\pi\rho$. Further the parameter of k^2 can have any positive value and starting from *0* onwards, so that the pressure is left undecided. It is interesting to note that pressure is given by Eq(72.4) (Eddington, 1924) in the case of anisotropic coordinate system and is given by the equation.

$$P = \left(\frac{\alpha}{8\pi}\right)\left[\frac{\frac{3}{2}(1-\alpha r^2)^{\frac{1}{2}} - \frac{3}{2}(1-\alpha a^2)^{\frac{1}{2}}}{\frac{3}{2}(1-\alpha a^2)^{\frac{1}{2}} - \frac{1}{2}(1-\alpha r^2)^{\frac{1}{2}}}\right] \quad (16)$$

Also Kelkar and Shrivastav (1999) have found the pressure according to Newton's theory. They have used the relation

$$V = \frac{2\pi\rho(3a^2 - r^2)}{3} \quad (17)$$

and

$$\frac{\partial V}{\partial r} = \frac{1}{\rho}\frac{\partial P}{\partial r} \quad (18)$$

giving

$$P = \frac{2\pi\rho^2(a^2 - r^2)}{3} \quad (19)$$

so that the pressure is given uniquely. Another interesting point in this regard is the fact that Einstein has converted the hydrodynamic equation $T^k_{i,k} = 0$ of the general relativity (GR) into an identity $T^k_{i,k} \equiv 0$. But Einstein's field equation for anisotropic line element can be solved (as will be shown in the papers to follow) without using the equation $T^k_{i,k} = 0$. If we put pressure $P = 0$ in the solution of the anisotropic case, we can still get the solution of field equation. Hence for anisotropic line element also there can be static sphere of dust according to Einstein. These anomalies obviously can be avoided by taking $T^k_{i,k} = 0$ as an independent equation of GR and making the field equation different from Einstein's equation. This has been done by Kelkar and Shrivastav (1999) by proposing two new field equations as

(i) $T^k_{i,k} = 0$ (20)

(ii) $R^k_i - \frac{1}{2}g^k_i R = 4\pi\rho_0 g^k_i + \eta^k_i$ (21)

where

$$\eta^{ik} = 4\pi P\left(4\frac{dx^i}{ds}\frac{dx^k}{ds} - g^{ik}\right) \quad (22)$$

so that $g^{ik}\eta_{ik} = 0$ and pressure is negligible as compared with proper density ρ_0. Hence

$$R^{ik} \cong -4\pi\rho_0 g^{ik} \quad (23)$$

and

$$R_{44} \cong -4\pi\rho_0 g_{44} \cong -4\pi\rho_0 \quad (24)$$

This brings about Newtonian approximation.

CONCLUSION

Einstein's theory predicts that there can be static sphere of dust considering both isotropic and anisotropic metric line element which means that the theory is not giving the results which gives Newtonian approximation in the case of rotation and radial motion of a star. We can get Newtonian approximation by considering $T^k_{i,k} = 0$ as a separate equation and modifying Einstein's equation as given in result and discussion.

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