Fundamental Journals
IJFPS
International Journal of Fundamental Physical Sciences
ISSN : 2231-8186
Full Length Research Paper
http://fundamentaljournals.org/ijfpss

A geometric representation for visualizing relativistic length and time measurement

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(*Received Nov 2011; Published Dec 2011*)

ABSTRACT

The simplest solution of Einstein's field equations is Schwarzschild solution. This solution is not able to describe for any non-spherical shaped objects. Some stars and galaxies are in ellipsoidal. Consequently, the gravitational field around these objects should be different in compare with spherical form. This paper is considering a new line element so that we are able to construct not only spherical objects, but also we are able to explain an ellipsoidal object too. This new line element is more accurate and complete than Schwarzschild line element. In this research we see that Schwarzschild line element and its solution is only a part of whole work, which we have done. For more consideration we applied this metric to an arbitrary object in the next step. Moreover, we used this line element for solution of a planetary orbit of an ellipsoid planet by using Einstein's field equations. These equations used for the exterior solution of an ellipsoidal celestial object.

Key words: General relativity, elliptical objects, planetary orbits

INTRODUCTION

The first inking of relativity for many people may come from any physics texts which introduce contraction of moving dimensions or slow downing of a moving clock as the most famous facts of theory of special relativity.

Majority of these physics texts don't deal with visualizing of relativistic concepts ,or if they do, it is always the one that uses different observation of a beam of light in a moving train according to two observers inside and outside the train who are in relative motion. Such explanations disregard the simple geometry aspect of the theory, namely covariance principle (Thorne, 1994).

Since relativity equation's can be explained geometrically, it is emphasized that special relativity must be approached sometimes from the point of view of geometry. Which leads us to a better understanding of the theory?

The Lorentz transformation equations transform an event in a reference frame in 4 dimensions (x, y, z, t) to a new frame which are in relative motion. The transformation coefficient that is the result of changing one coordinate system to anther is a unique transformation which is called Lorentz transformation. To simplify the equations of the transformation, all dimensions are eliminated except one space and the time dimension.

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \;,\; x' = \gamma(x - vt) \;,\quad y' = y \;,\; z' = z \tag{1}$$

Where,

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{2}$$

The natural resultant of the transformation is length contraction and/or time dilation. In this case the coefficient γ plays an important role. There are not lots of student who can understand the interesting result and the idea behind of this transformation system. The difficulty is not because of the mathematics, because this is a liner and relatively a simple equation. The most confusing fact is generalized transformation and applies this to explain length contraction and time dilation. The difficulty of understanding two reference frames through a single equation is related to inherent tendency of classical way of our thinking. The fact that special relativity is based on a relativistic idea, results in relativistic understanding of the entire involved concepts. And yet there is no way to avoid this confusion without abandoning use of mentioned example (different observation of a beam of light in a stationary and/or moving train according to an observer outside the train).

However there is a way to avoid the confession and that is, to use in its place geometric. One of some geometrical representation of Lorentz transformation is by a presentation which takes advantage of equations:

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} \qquad (3)$$

$$t_0 = t\sqrt{1-\frac{v^2}{c^2}} \qquad (4)$$

In order to use just trigonometry relation to visualize the equation 2 on a trigonometry circle we rearrange equation (3) to

$$L = L_0 cos\theta \qquad (5)$$

$$t_0 = t\ cos\theta \qquad (6)$$

where,

$$\sqrt{1-\frac{v^2}{c^2}} = cos\theta \qquad (7)$$

According to the following equation,

$$\sqrt{1-sin^2\theta} = cos\theta \qquad (8)$$

we obtain,

$$sin\theta = v/c \qquad (9)$$

These equations imply the possibility of visualizing time dilation and length contraction by mean of trigonometric relationships on a trigonometric semicircle. The disadvantage of this presentation is that two reference frames are not being shown, and the device is always assumed as moving reference frame. But the advantage of this is the length contraction and time dilation can be easily visualized and understood. In this paper, two geometric representations are proposed for interpretation and measurement of relativistic dimensions and time. Moreover, this approach can simplify explanation of lengths contraction and time dilation and overall related parameters in special relativity.

THEORY:

A GEOMETRIC REPRESENTATION FOR MEASURING RELATIVISTIC DIMENSIONS

If a ruler with length of L_0 moves with relative velocity, $v/$c, according to especial relativity equations, it will be observed shorter with length L.

$$\frac{L}{L_0} = \sqrt{1-\frac{v^2}{c^2}} \qquad (8)$$

Since $v/c < 1$, so there is a unique θ in the interval of $0 \le \theta \le \pi$ that gives $|v/\mathrm{c}| = \sin(\theta/2)$, therefore;

$$L = L_0 \cos(\theta/2).$$

Now, if plot a half circle with diameter of Lo and separate an arc with angle θ, so that $< BOC = \theta$, easily it consequences that (Figure 1); $AC/AB = \cos(\theta/2)$ and $AB = L_0$ and $AC = L$.

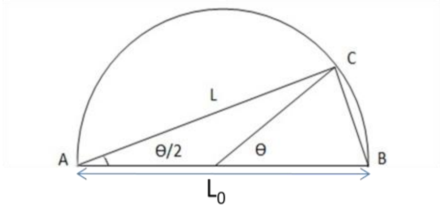

Figure.1. Geometric presentation for relativistic dimension

Based on the Figure.1, and its interpretation as a protractor, the length of stationary ruler is the diameter of the semi circle which is L_0. Obviously, the tool is considered to have diameter of $L_0 = 1$ for simplicity. To determine the length of the ruler, L, when it moves with velocity v, first find $sin\theta/2 = v/c$ to indicate the point C on the semicircle. Then the chord that is drawn from point A to point C makes angle $\theta/2$ with the diameter. The chord AC is the length of the contracted ruler; $L = L_0 cos\theta/2$. On this way the dimension of a moving object can be found geometrically on the given semi-circle by scaling this tool. To calibrate the tool, the semicircle should be scaled in terms of velocity. This means the unit of the scale is a multiplication to the velocity of light ($3 \times 10^8\ m/s$). Each number on the scale shows the relative speed of object times c. here two relativistic lengths are measured:

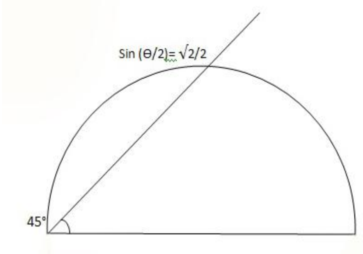

Figure. 2. Length of a ruler moves with speed of $\sqrt{2}/2c$.

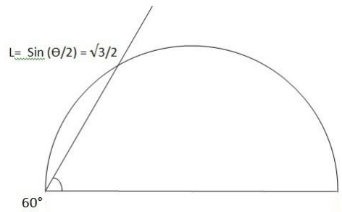

Figure 3. Length of a ruler moving with speed of $\frac{1}{2}c$.

GEOMETRIC REPRESENTATION, MEASURING RELATIVISTIC TIME

In this section a tool for measuring relativistic time is proposed that can facilitate understanding the time delay phenomenon in the theory. An example can show the situation to clarify how difficult it is. To do so, first we try to distinguish between this absolute principle and a controversial method to define time by giving an example: Assume there are two observers, A and B, which are located at the same distance from you (O). You send a signal to each one of them, simultaneously. The observer A starts moving with high speed, v, at the exact moment when O sends the signal to A and B. But observer B is remaining stationary at its location. The observer O is expecting that A and B would receive the signals instantly. But the observer A receives the signal later than B. This is related to the fact that v is comparable to the speed of light; therefore, the photon that carries the signal will need longer time for travelling to reach to A. The difference of travelling time of photon that A and B feel is related to the relative speed of observer A. This means the time for observer A is depending on its relative speed to C; e.g. v/c. So, the concept of time is not absolute parameter, but a relativistic parameter that depends on relative speed to c. This is the main idea of time in relativity theory. On the other hand, the time delay of receiving signal by observer A is a function of the v/c and it can change by varying this relativistic speed. In the other words, as much as v is closer to c, the delay time is greater. It can be interpreted by O on this way: In the frame of the observer A, time flows slower than the time flow in the frame of observer B. Here we concern three important points:

1-This model is a geometric representation to give a better understanding of especial relativity so, this model is not necessarily gives the real definition of time. 2- However, the model should persist on special relativity and its equation, and also agrees with all parameters in it. 3- This model also must have capability to predict, accepting of time as the fourth dimension.

According to the above discussion, we assume that the concept of passing time is related to travelling of light, which is a row of separated and countable photons from a point to another, in space. Assume an observer 'A' that is standing at a point in space and equipped with a photonic needle. A photonic needle is a needle with a tip which has the size of a photon. The observer can measure the passing time by counting the numbers of photons passing by the tip of the needle. This is the meaning of time measurement in this model. Two points of special relativity theory are needed to give a complete picture of passing time in this model.

1. THE CONSTANCY OF SPEED OF LIGHT IN ALL REFERENCE FRAMES

First we have to define the constantly of c base on my model. As we mentioned, counting the number of photons generated from an event that reaches to a point of space (e.g. observer) is a measure of passing time understood by the observer from that particular event. When an observer is stationary, he counts certain number of photons passing from his photonic needle, which we accept it as a measure of speed of light which is constant. So, two stationary observers will count the same number of photons in an equal period of time, which is representing c. But a moving observer needs different time to measure the same amount of photons. This means an observer which is moving with relative speed of $v/c = 0.5$, to the same direction but going away, needs two times longer time to count the same number of photons. In other words, if the relative speed of an observer in the same direction with the coming signal is $0.5c$, it means the time is passing for the observer two times slower, so he need two times longer time to compensate the deficiency of number of measured photons $v = 0$.

2. A STATIONARY OBSERVER COUNTS THE NUMBER OF PHOTONS THAT REACHES TO ANOTHER MOVING OBSERVER WITH RELATIVE SPEED OF v/c, DIFFERENTLY; i.e. MEASURES THE PASSING TIME, DIFFERENTLY.

Assume an observer is moving with a high velocity and a stationary observer in reference frame tries to measure the passing time in the moving frame. Observer in reference frame sends a beam of photons towards him and tries to count the number of photons which pass by the tip photonic needle of the moving observer. If the moving frame has speed of c, stationary observer will realize that no photon can pass by the tip of the needle of moving observer. Therefore, the moving observer never will receive the signal, as if the event has not occurred at all. The stationary observer will understand it as the time was stopped for the moving observer, because the moving observer cannot count any photons. Since the moving observer needs infinite time (∞) to count photons of the signal, he cannot feel time in his frame, as if the event has not occurred. In this case, the stationary observer cannot distinguish the motion of moving observer, since time is stopped in the moving frame (Figure 4). It is in contradiction with the constancy of speed of light. Therefore, the relativistic time should be considered to explain the situation.

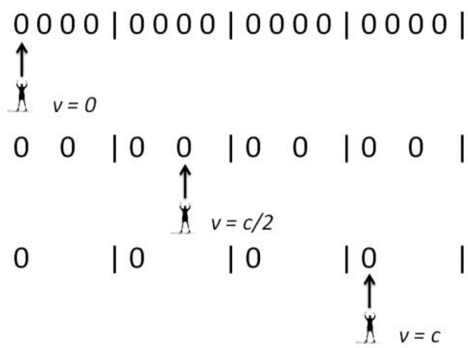

Figure4. Different displacement of observers respect to photons

The stationary observer can solve the contradiction by considering time term in the equation (9) to distinguish between the stationary situation due to stopped time and stationary case due to non -existing movement. A signal of light in a period of time (dt), travels the distance of cdt which is equal to the distance $\overrightarrow{dr} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}$. So, the difference between them is $ds = 0$. The equation (2) shows the difference between $(cdt)^2$ and $(dr)^2$. In other words, $(dr)^2$ is the difference between distances travelled by a photon and an observer. If the speed of the observer is maximum (c), then $\Delta r = c\Delta t$ and ΔS is minimum $\Delta S = 0$. This means the passing time is stopped. If the speed of the moving observer is minimum (i.e. stationary observer $v = 0$) then the difference is maximum $\Delta S = c\Delta t$. This means the passing time is not relativistic.

$$\Delta s^2 = \Delta r^2 - (c\Delta t)^2 \tag{9}$$

So we can understand the importance of considering time as the fourth dimension. Finally, it is needed to find the trajectory of a moving observer. Assuming the speed of an observer is c, the time is stopped in his frame, so there is no room for time; therefore, this observer seems stationary from the view of a stationary observer. In the case when speed of observer less than C, the time can be felt by a stationary observer. The both observers, with $v/C = 1$ and $v/C < 1$, should measure the speed of light similarly equal to C based on the postulate. To satisfy these two cases, the photonic needle tip should have a curvature trajectory in order to count the same number of photons in both cases. In Figure 1, the position of point C relates the relativistic v/C and L as shown above. If $v = c$ the point C is located at A ($L = 0$ and $V/C = 1$), while if the $V/C < 1$, the point C is located somewhere between A and B. This means that the observer should move in a circular curvature trajectory. The actual meaning of semi circle of Figure 5 and Figure 6 is this circular curvature trajectory that the observer should move in

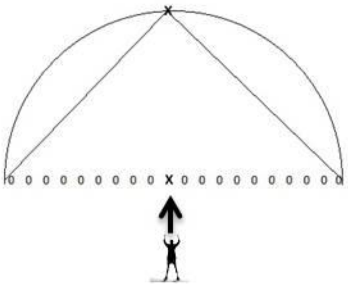

Figure5. Shows the curvature trajectory with $v = (1/2)c$

REFRENCES

Black holes and time warps: Einstein's outrageous legacy.
Introduction to Special Relativity, 2nd ed. Wolfgang Rindler

Thorne, K. S. (1994). *Black holes and time warps: Einstein's outrageous legacy*: WW Norton & Company.

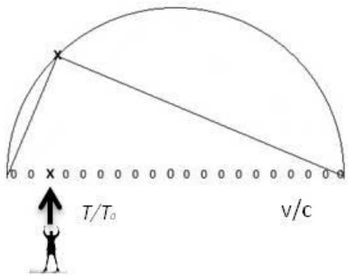

Figure6. Shows the curvature trajectory with $v = (1/3)c$

CONCLUSION

The fact that this geometric presentation persist on the equation of length contraction, shows that this presentation is used to visualized relativistic length and time correctly. The advantage of geometry is that physical situations are presented as seen by one observer, and the problem is how this situation will appear for different observer. By using this geometric device, contraction and dilation facts are visualized clearly. On this way, student will try to reduce the complicated facts of the theory to a geometric visualization.

GregorLNaber,
The GeometryofMinkowski Space time